Teaching Script

1. IDENTITY OF TEACHING SCENARIO

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Thematic of the teaching scenario: Euclidean Geometry - Parallelogram

Level – Class: 15-16 years old / 1st High School (10^{th} Grade)

Recommended Teaching Hours: Three (3)

Expected Learning Outcomes

Students should:

- Define and distinguish between convex and non-convex quadrilaterals.
- Define, recognize and classify quadrilaterals into parallelograms and non-parallelograms.
- Prove the basic properties and criteria of the parallelogram and distinguish which of them characterize it.
- Prove whether a quadrilateral is or is not a parallelogram based on given information about its sides and angles.
- Make use of the definition and criteria in order to draw parallelograms that will keep their properties unchanged after dynamic changes in their elements, either in a dynamic geometry environment or by using geometric instruments.
- To acquire the ability to understand, judge, create and use mathematics correctly in a variety of mathematical contexts and situations in which mathematics plays or could play a role and thus function critically.

Prerequisite student capabilities (cognitive and socio-cultural)

Students in Middle School (Gymnasium) have been exposed to concepts related to the concept of parallelogram, where they have explored types of quadrilaterals, formulated definitions and classified them based on their properties. Also using geometric instruments and digital tools they formulated conjectures related to the properties of parallelograms and drew a parallelogram. They also know the triangle congruence criteria. Also in the 10^{th} Grade, the following modules with corresponding Expected Learning Outcomes have preceded:

- Angle relationships formed by two parallel lines intersected by a third line.
- Sum of angles of convex polygons.
- Triangle equality criteria.

2. TEACHING FRAMEWORK AND MANAGEMENT

About student and learning

As with any other mathematical concept, so with the concepts of geometric space and in this particular case with the concept of the parallelogram, it is important that the teaching begins and evolves with a trigger from the real world with the aim of highlighting the intuitive point of view of the children. Specifically, taking for granted that children's mathematical/geometrical thinking develops in successive levels (Van Hiele levels of Geometrical thinking), the teaching of the parallelogram in this project is adapted and follows the following steps:

- The visualization/distinction of the parallelogram through a set of convex quadrilaterals and its identification/description, possibly with a limited geometric vocabulary.
- The identification of the constituent elements of the parallelogram, the description of their properties with the gradual development of the appropriate vocabulary. Children's investigation at this step is mainly empirical in nature without necessarily making formal definitions and logical proofs.
- The logical arrangement of the properties of parallelograms, the clarification of the role of the definition and criteria in comparison with its properties. In this step, the relationships of sides, angles, diagonals and how they characterize or not the quadrilateral as a parallelogram become the object of research.
- Reproducing or synthesizing proofs, using logical reasoning, discovering relationships and making generalizations (some Big Ideas of Mathematics). In this step the student invokes the role of definition and criteria as the sufficient and necessary conditions to characterize a four-sided parallelogram and utilize the appropriate proof methods.
- Abstract reasoning, the wider application of what was demonstrated in the previous steps to real problems in which students are asked to discover parallelograms and use their knowledge on them or even enter the logic of the geometric constructions in which this knowledge is integral element.

In particular, the main difficulties of the students and the critical points, according to research, which are expected to be addressed are:

- The distinction between definition, properties and criteria of parallelograms.
- The misconception that the diagonal of a parallelogram is also the bisector of the corresponding angle.
- The correct verbal formulation of properties and criteria.
- The rigorous proof of properties and criteria.
- Finding suitable procedures for constructing parallelograms with ruler and compass using all criteria.

Characteristics of the mathematical activity that is sought to emerge during the students' engagement with each of the specific projects.

- The essence of this didactic proposal is training in the use of criteria as tools for locating parallelograms and not the sterile memorization and reproduction of them at a theoretical level. Through appropriate activities, they are invited to identify the differences in criteria and properties by experimenting either with hand-held material, with constructions on paper or with dynamic quadrilaterals, through which the geometric design (image) is distinguished from the geometric shape (class of designs with common characteristics) and misconceptions are visualized, one of the crucial points of teaching.
- The correct verbal formulation of definitions, properties and criteria occurs in this phase and can be achieved by all students, a fact that develops self-confidence and creates positive motivation for the continuation.
- The students are invited to collaboratively /establish their reasoning based on already known propositions (criteria of equality of triangles, sum of angles of a polygon, relations of angles formed by two parallel lines intersected by a third line), to investigate their options and conclusions, to interpret and modify their choices by checking and reassessing as many times as necessary their every move.
- The transition from geometric relationships to problem solving with a real background is engaging and achieved through dynamic activity. Thus, they come into contact with ways in which parallelograms can be used, exploiting their properties and criteria, in order to acquire an additional resource in the context of their mathematical literacy.

Teaching actions – Teaching practices

The role of the teacher

The teacher plans the course of the teaching and as the orchestrator of the whole process he makes sure, together with the worksheet, to give the first necessary practical instructions (e.g. use of software or hands-on material) to facilitate and mediate so that the groups and the role of each child in it to be defined harmoniously and with the consent of the participants.

In this first phase of the organization, he explains the course of action and prompts the students to follow the instructions on the worksheet.

He then subtly observes the students' work, records practices and behaviors moving from group to group and answers questions posed without suggesting solutions but taking care to create a supportive learning environment in which the students themselves through experimentation and dialogic confrontations, they will review, evaluate, judge and ultimately make decisions and answer their own questions.

Coordinates the dialogue between groups while at the same time being able to use students' "mistakes" or original ideas to ask additional questions, seeking to connect and integrate new knowledge with pre-existing knowledge or make extensions that will prepare the ground for later knowledge.

Encourages the active involvement of all students in either individual or group activities utilizing the principles of educational psychology and the practice of multiple approaches and listens to student concerns that may be related to social or cultural differences by suggesting role changes or giving the group alternatives to a logic of its self-regulation.

In the final stage of the evaluation, he makes sure to highlight any effort even of the weakest students, to give the necessary feedback avoiding the categorization of students, to reward participation and cooperation and to applaud progress and not necessarily excellence.

The role of students

In some activities, the class can be divided into 4 or 5 groups of 4 or 5 people, whose roles are formed by choosing the participants. One of the children in each group handles the computer and the Geogebra files, two of them deal with the paper activities with a ruler and ruler, and the other two with the hands-on material (cards to cut or fold and investigate the equality of triangles with displacement, application and identification).

Alternatively, one or two groups may work with Geogebra with one operator taking turns and collaborating on their actions and conclusions, one or two groups all working with paper, ruler and compass, or other geometric instruments such as ruler and protractor for measurements and guesswork and a group with the carpentry material.

The worksheet is given to everyone and completed after discussion and interaction of the differentiated approaches.

At the end of each Project, the groups are invited to compare their results, documenting the conclusions damn them.

A set of Projects from the textbook or any other resource is also assigned as individual homework.

Managing classroom dynamics

In the specific projects, students work in groups and are asked individually and as a group to find or discover:

- either a way to calculate the demand by making a series of designs
- either a way to compare shapes and their corresponding properties, applying an appropriate method (equality of triangles, identification, etc.)
- either create and support their own methods if they choose to work exploratorily outside the standard framework with additional tools (e.g tests).
- To cooperate while respecting the opinion of others, to discuss and reach a common text.
- Highlight what they consider important by describing the methods they followed (this is also a helpful element for the teacher's observation of processes and conclusions).

In working groups, students work out the problem and discuss methods of dealing with it, through dialogue and argumentation.

The activities concern:

a) in the identification of the requested problem,

b) in the choice of method,

c) in processing the necessary designs,

d) checking the validity of the result and taking corrective action accordingly, if necessary.

Managing practical parameters (such as time and logistics)
The scenario can be implemented either in the IT Lab or in a classroom. In terms of space management, the groups are placed so that those working with Geogebra have good visual contact with the computer screen and space comfort for those working with geometric instruments or haptic material.
Regarding the distribution of time, in the 1st teaching hour the introduction part and projects 1, 2 will be implemented, in the 2nd teaching hour we propose to implement Projects 3, 4, 5, 6, 7, 8, 9 and the 3rd teaching hour time we propose to follow the final evaluation with Projects 10 and 11, the plenary discussion and the feedback.

3. EVALUATION

Assessment for learning and feedback of teaching work and how to check and see what each student and the class has achieved collectively.

During the teaching, the students will be evaluated based on the worksheet, the cooperation and communication between the members of the group and the plenary session during the discussion about the results of the mathematical activities they developed during their engagement with the Projects.

After the teaching of the 3rd lesson there will be a final evaluation.

For the teaching project: How will it be checked / established whether the teaching options were successful or ineffective?

In order to evaluate the results of the teaching work and to check its validity and the optimal choice of methods, the teacher during the course of teaching observes and records whether:

- Students relate the results to what they have been taught.
- Students relate, evaluating, inferences about quantities in terms of their validity and consistency.
- Detects and corrects errors autonomously or, related to the above, accepts the rationality of verification when based on established knowledge invoked by peers or the teacher.
- He justifies the methodological/resolution choices he made.
- Expresses verbally the course of resolution he performed.
- Can teach the entire solution process to his classmates.

In addition, upon completion of the three-hour lesson, students are asked to complete the following self-evaluation sheet.

	CRITERIA	NOT AT ALL	A LITTLE	ENOUGH	A LOT
1	I was able to verbalize what was requested.				
2	I was able to locate the appropriate parallelogram property.				
3	I correctly applied the criteria to the projects.				
4	I was able to tell the difference between the properties of a parallelogram and the criteria for a parallelogram.				
5	I used the "new knowledge" to solve the problem or build.				
6	I encountered problems in the paper design of some requested projects.				
7	I had problems handling the software.				
8	I had problems handling the handpiece.				
9	The instructions given to me were clear and understandable.				
10	I participated in the group actively.				
11	The team helped me realize wrong choices and correct them.				
12	My participation in the group helped some of my classmates to understand and improve.				
13	By participating in the group, I better understood the concept of parallelogram.				
14	My professor encouraged me to use tools assigned to me.				
15	My professor was helpful in solving the problems I faced.				

4. FEEDBACK

➢ **For teaching planning:**

Given that the design of the teaching in the mathematical projects was done in the context of multiple entry points, with an escalating degree of difficulty and with the possibility of different approaches that the students can develop in the same project (digital tool, drawing shapes with geometric instruments, use of hands-on material) the teacher (and possibly another experienced observer) should answer the following questions:

- Were the projects addressed to all students and if so, did they enable them to achieve the objectives of the activities to a lesser or greater extent?
- Did the use of the software either individually or collaboratively within groups attract and maintain at a satisfactory level the attention of even the usually indifferent students by stimulating their curiosity, provoking and promoting competition?
- Was there familiarity with handling the software, a basic condition for teaching to be conducted without delays, or is it necessary to spend time before teaching to practice on the software?
- Was the use of sociocultural tools, such as language, symbols, texts, effective?
- Was the use of hands-on material useful, necessary or helpful and in which of the projects did it work positively in visualizing and understanding what was requested?
- Was the presentation of the instructions understandable to the students?
- Did working in groups help to overcome points where there were questions?

➢ **For the learning process:**

In agreement and continuation of the above planning, questions that may concern the teacher regarding the learning process are:

- Did the immediate and in some cases corrective feedback offered by the software result in the students discovering on their own and therefore realizing their mistakes but also engaging in reflection processes in order to be guided in the right direction?
- Was the teacher's role limited to a facilitator with frequently asked questions to provide feedback and control the learning process?
- In discussion-questions that followed the teachings, the students answered with clear references to activities assigned to them and to what percentage this fact allows us to conclude that practical engagement, the image and the interaction helps to more easily capture definitions and conclusions?
- Lack of discipline is also a factor that affects the quality of teaching in the laboratory much more than in the classroom. Therefore, does the teaching in the laboratory require a small number of students (probably no more than ten people) to ensure that the software is used without problems and that the control by the teacher is sufficient and effective?

➢ **For the teaching approach:**

- Was there an increase in student interest?
- Did they feel the lesson became more engaging?
- Did the children's mistakes that were purposefully treated as an integral part of the learning process form a springboard for understanding the mathematical content by the children themselves?
- Incorrect designs on the paper or misunderstandings and incomplete application of the criteria were points of reflection and re-examination of the methods used by the students approaching the question in a variety of ways, a fact that led to the essential understanding?
- Given that peer feedback has a number of advantages for students providing the feedback as well as those receiving it, group work led to processes of reflection on learning and enhanced students' understanding of their own learning ; (metacognition)
- The use of new technologies during the teaching process creates particularities and requires flexible teaching schemes and techniques. Based on this, can it be considered that the variety of approaches chosen is judged positively in terms of the final achievement of the teaching objectives?

➢ **For the feedback of the teacher and his practice (professional development):**

After the end of the teaching, the teacher is invited to:

- To evaluate the mathematical projects he used, how they created the corresponding mathematical activity and whether it managed to highlight "useful" mathematics.
- To distinguish the characteristics of digital tools which made them useful in their use in the teaching practice.
- To distinguish the characteristics of the hands-on material which made it useful in its use in the teaching practice.
- To argue how each approach or each project (with or without the use of digital tools or handiwork) and the corresponding activity, helped to construct meanings in alternative ways and why the variety of approaches was important.
- To argue how each approach or each project (with or without the use of digital tools or hands-on material) helped to design the learning environment proposed by the Curriculum - individual construction of knowledge, exploratory/discovery learning, problem solving, participatory/ cooperative learning.
- To be concerned with unexpected incidents by answering questions like "what new has arisen" and to give alternative ways of approaching.
- To question or even redefine strategies of inclusion and differentiation, adapting the mathematical projects to the previous knowledge, experiences, culture and the pace that the students learn, if he judges that these did not work.
- To improve, revise and repeat the teaching based on the data collected.

Teaching Process

1st teaching hour: Introduction

Brief information on Euclid's life and work and resource websites are provided for those interested in researching it further.

Euclid (flourished about 300 BC, Alexandria, Egypt):

The most prominent mathematician of Greco-Roman antiquity, known for his treatise on geometry, the Elements (13 books). Euclid understood that building a logical and rigorous geometry (and mathematics) depends on foundations. Euclid began Book I with 23 definitions (such as "a point is that which has no part" and "a line is length without breadth"), five unprovable postulates which Euclid called axioms, and five further unprovable postulates which he called common concepts(notions).

Euclid's postulates (Axioms)

1. Given two points there is a straight line joining them.
2. A straight-line segment can be extended indefinitely.
3. A circle can be constructed when a point for its center and a distance for its radius are given.
4. All right angles are equal.
5. If a straight line intersects two other straight lines so that two angles within and on those parts have a sum less than two right angles, then the two lines intersect in that half-plane in which the angles have a sum less than two right angles.

Common concepts of Euclid

6. Things equal to the same thing are equal.
7. If equals are added to equals, the totals are equal.
8. If equals are subtracted from equals, the remainders are equal.
9. Things that coincide with each other are equal.
10. The whole is greater than a part.

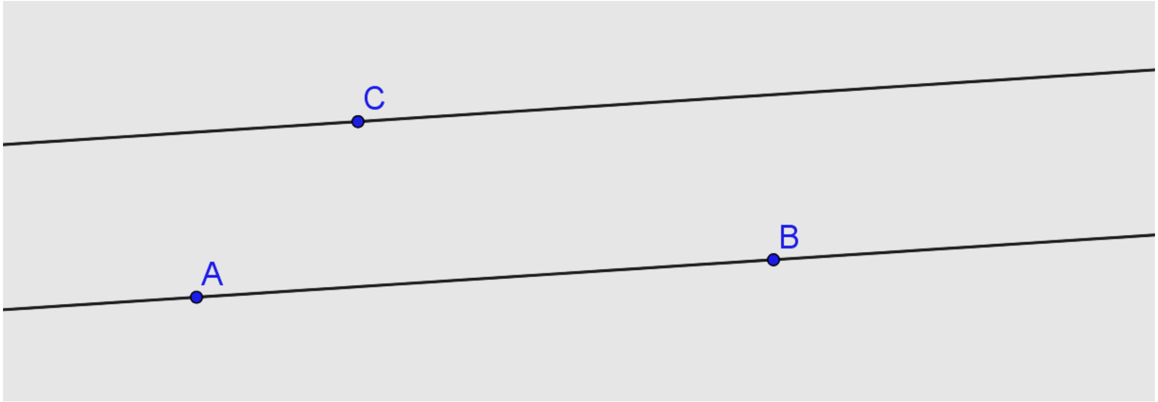
The 5th postulate and some of its consequences

Euclidean Geometry actually starts from this point, because after Euclid demanded that the 5th Axiom be accepted without proof, many other theorems were proved using it directly or indirectly and he built his Geometry. There are other Geometries who did not accept the 5th axiom but accepted another axiom instead as Euclid did. We call them non-Euclidean Geometries and they proved very different theorems than the Theorems of Euclidean Geometry.

• **We formulate two versions of the 5th axiom and describe it graphically.**

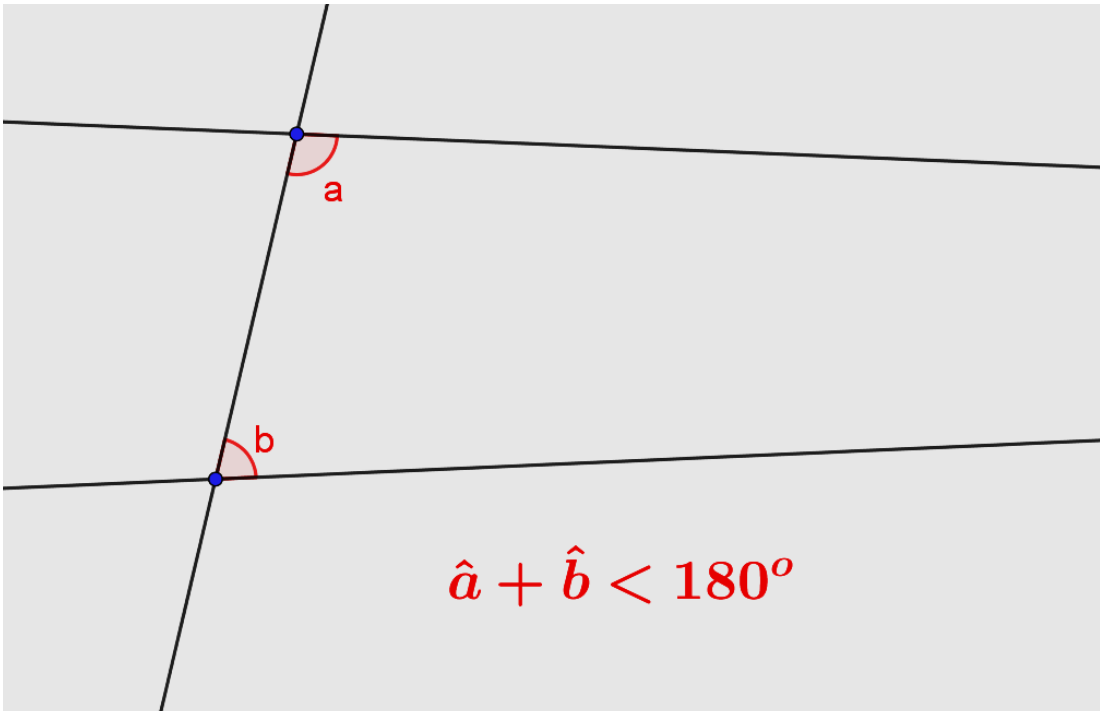
The 5th Axiom (Euclidean Demand/Request)

"For any given point not on a given line, there is exactly one line at the point that does not meet the given line (parallel line)."



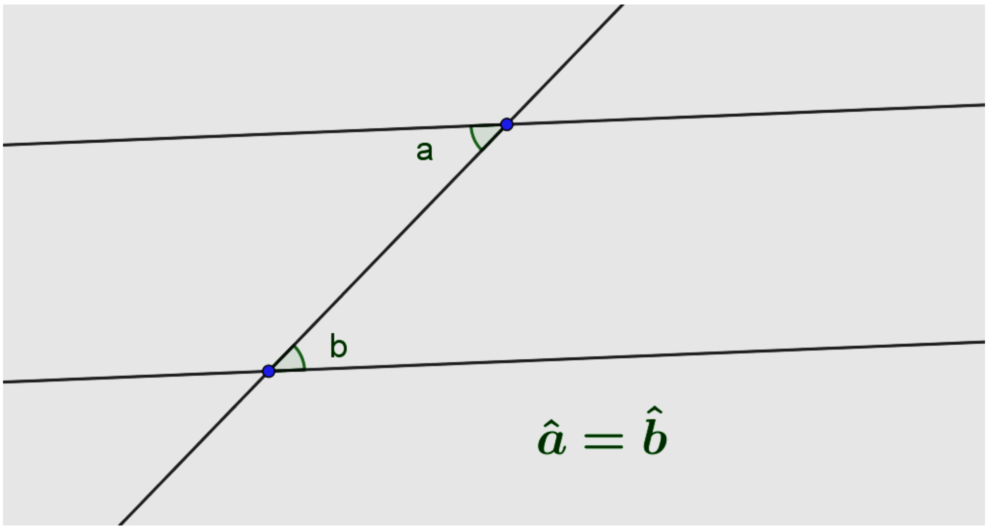
An equivalent 5th axiom instead of the previous one

"If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles".

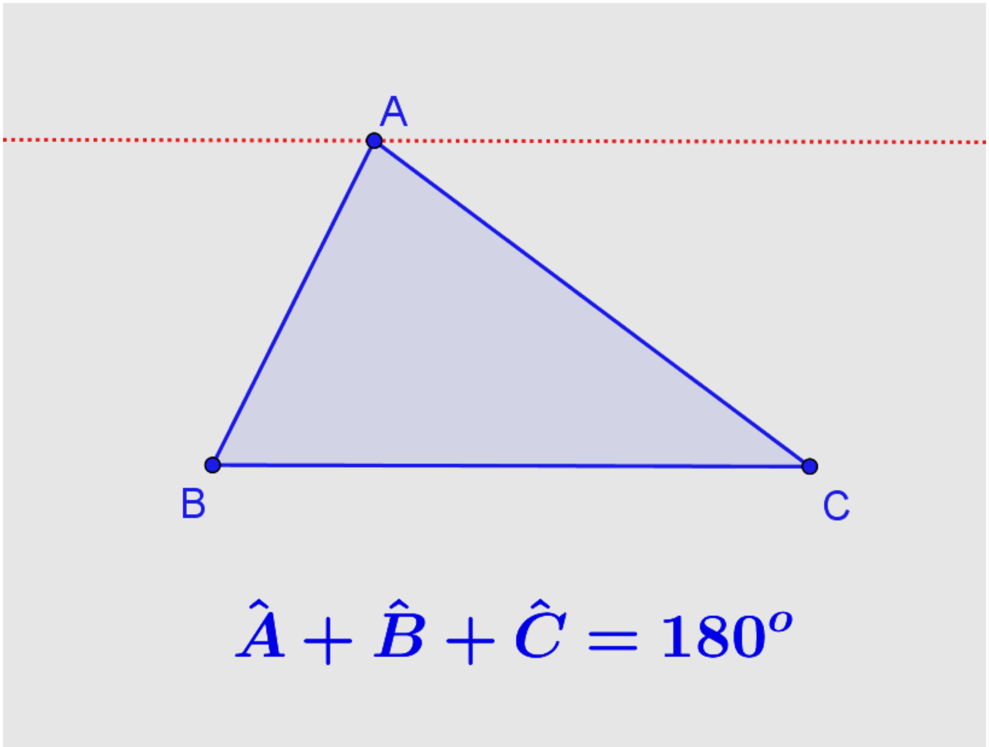


Finally, we prove some Theorems using the 5th Axiom stressing again that these are theorems of Euclidean Geometry and not necessarily of all other Geometries that have been constructed.

- ***"If two parallel lines are intersected by a third line then the interior alternate angles formed will be equal".***



- **"The sum of the angles of any triangle is two right angles (180^0)".**



Projects (and with possible extensions)

A suitable worksheet consists of:

- Parallelogram drawing projects that retain their properties after dynamically changing their elements in a dynamic geometry environment (Geogebra) and parallelogram drawing projects with ruler and compass.
- Projects that require the creation of conjectures and their substantiation with proof.

In addition, the teacher supplies each group with the necessary hands-on or digital material to approach some of the activities of the worksheet in alternative ways so that at the same time there is the necessary differentiation in teaching to meet the needs of each student.

TASK 1 (answer on the dotted lines)

1. Of the figures below, three differ from the rest in terms of the number of their sides.

………

2. Of the following quadrilaterals, one differs, it is not ……………., i.e. ………

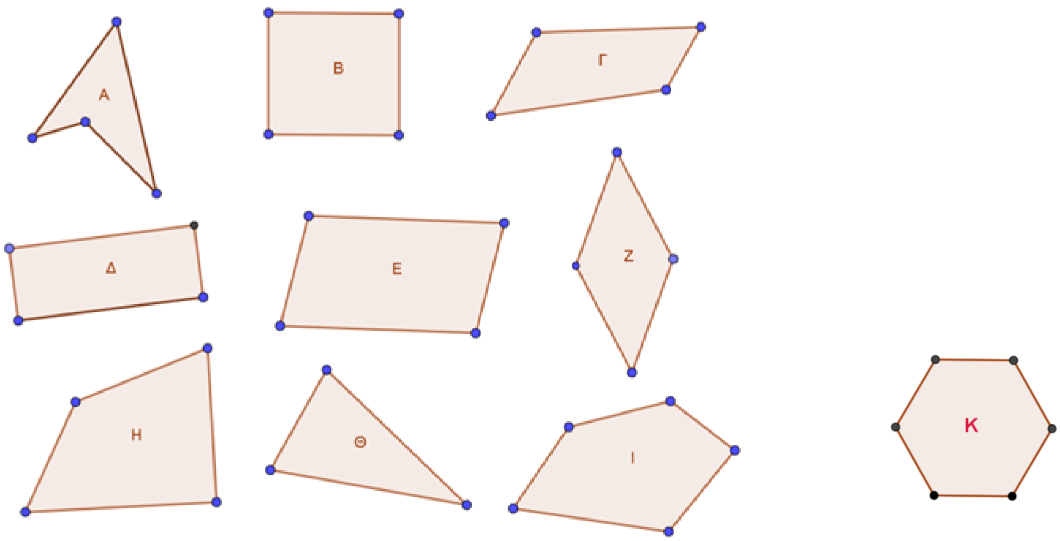
3. Of the following convex quadrilaterals, four have a common property:

……….

4. Definition: A parallelogram is called ……………………………………………………………………….

………

5. Is shape C a parallelogram? ……………………………………………………………………………………



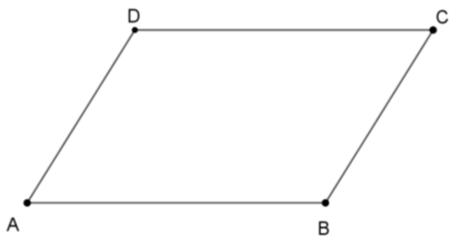
TASK 2: We prove the properties and criteria of parallelograms

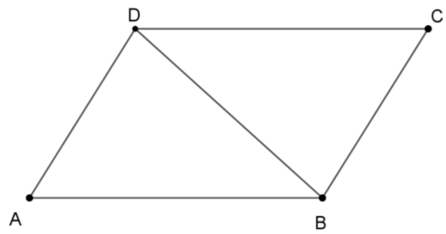
Definition: "A quadrilateral whose opposite sides are parallel is called a parallelogram."

Properties of parallelograms

1. The opposite sides of a parallelogram are equal.
2. The opposite angles of a parallelogram are equal.
3. The diagonals of a parallelogram bisect each other.

Proof

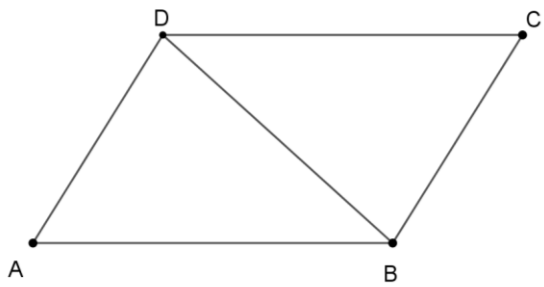


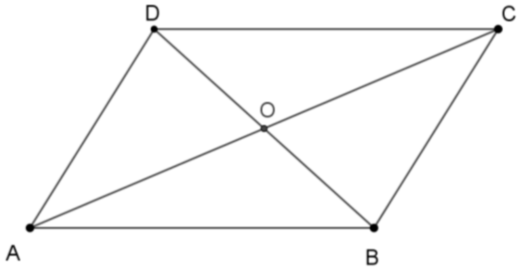


Criterion of parallelograms (Methods to prove that a quadrilateral is a parallelogram)

1. If a quadrilateral has its opposite sides parallel then it is a parallelogram.
2. If a convex quadrilateral has its opposite sides equal then it is a parallelogram.
3. If a convex quadrilateral has its opposite angles equal then it is a parallelogram.
4. If a convex quadrilateral has two opposite sides equal and parallel then it is a parallelogram.
5. If the diagonals of a convex quadrilateral bisect each other then it is a parallelogram.

Proof

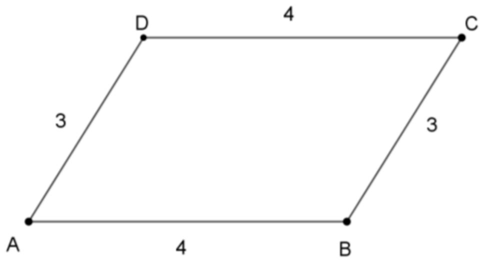


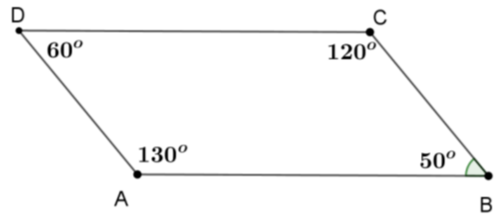


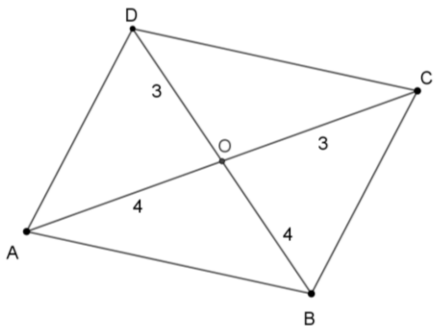
2nd teaching hour: Definitions, Properties, Criteria

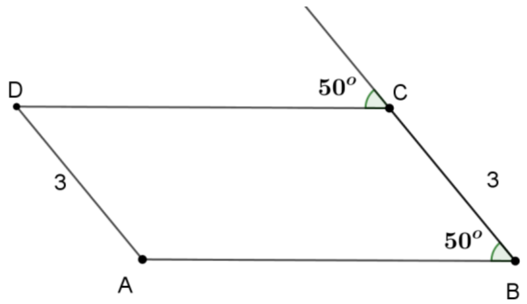
TASK 3

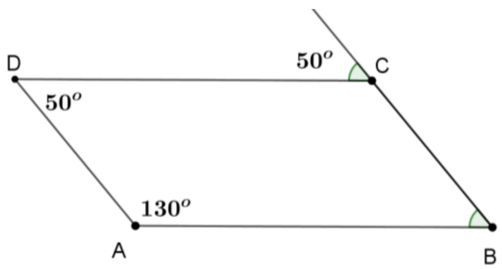
Check if the follow quadrilaterals are parallelograms, if not, or if we are not sure if they are parallelograms or not. Explain your answers.

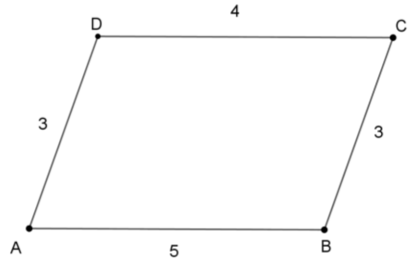


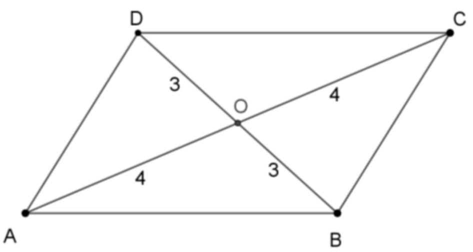


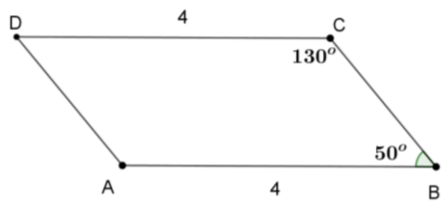






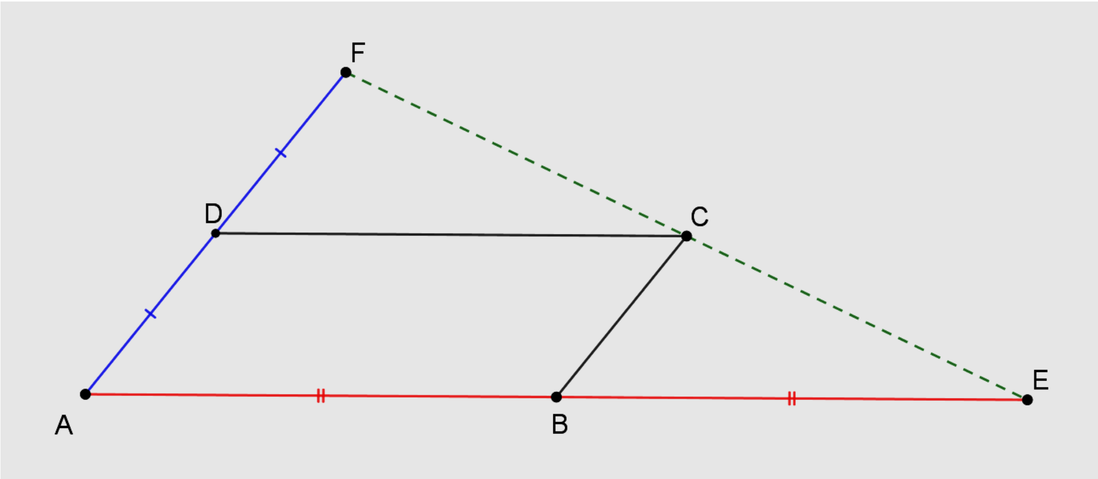






TASK 4

Let ABCD be a parallelogram and the extensions of sides AB and AD such that BE=AB and DF=AD. Prove that points E, C, F are on the same line using the 5th, Axiom. (Suggestion: With a suitable auxiliary line create parallelograms)



TASK 5

Given two segments AC and BD as diagonals construct a parallelogram ABCD.

Possible extensions: The activity can be used after and in the teaching of special parallelograms by investigating the relationships and positions of the diagonals of the formed parallelogram.

TASK 6: Two basic constructions of parallelograms using only the uncounted ruler and compass.

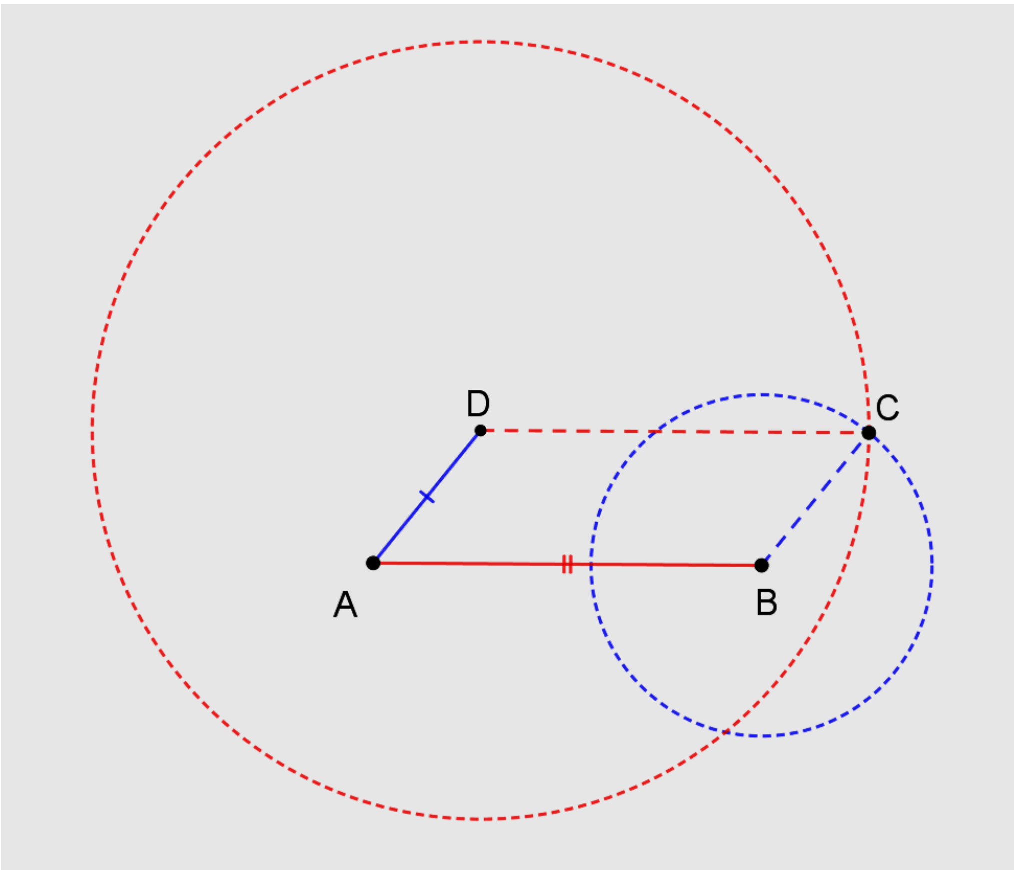
(We remind that Euclidean Geometry accepts only these types of constructions)

Construction with the use of Geogebra (Software of Dynamic Geometry)

1st Construction

- Construct two segments AB and AD
- Construct two circles (center B, radius AD) and (center D, radius AB)
- Name C the one section of the circles
- Prove that ABCD is a parallelogram

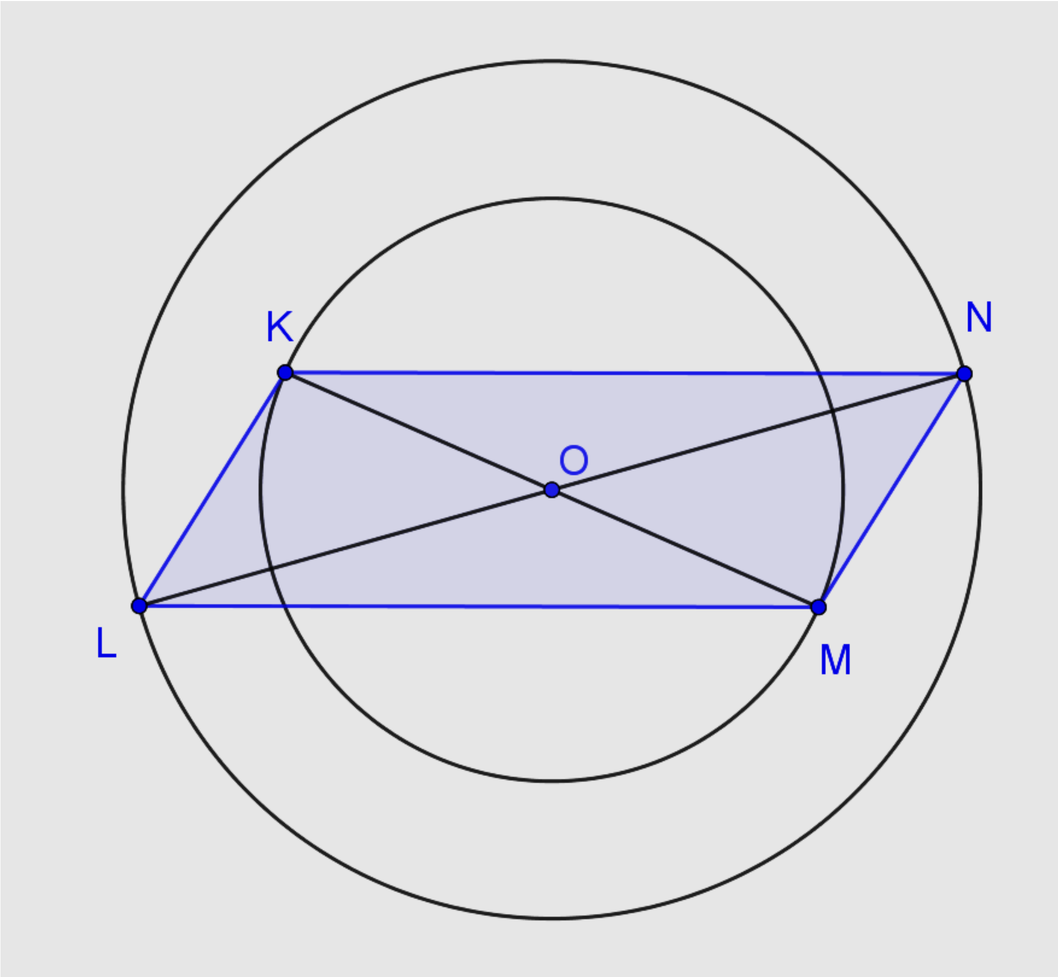
Screenshot of the Construction



2nd Construction

- Construct two circles with the same center
- Construct one diameter in each of these circles (not in the same straight line)
- Construct the convex quadrilateral defined by the endpoints of these diameters
- Prove that it is a parallelogram

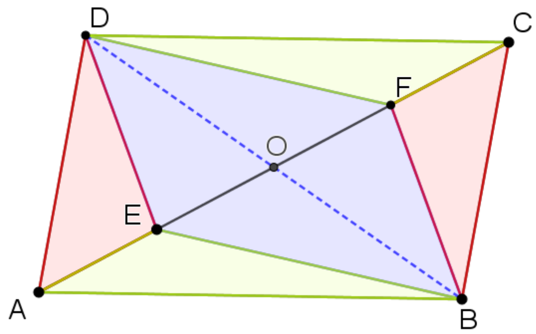
Screenshot of the Construction



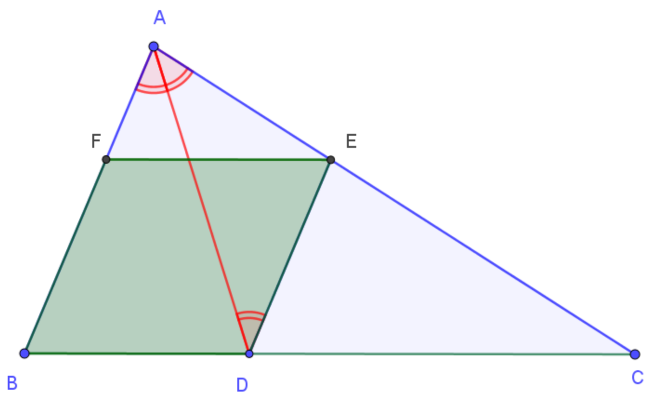
Comment: We can change the order of the steps of this construction as follows.

- Construct a line segment KM and find its midpoint O, using the segment perpendicular bisector construction,
- Construct a circle with center O
- Construct a diameter LN of this circle (not in the same straight line with KM)
- Prove that KLMN is a parallelogram

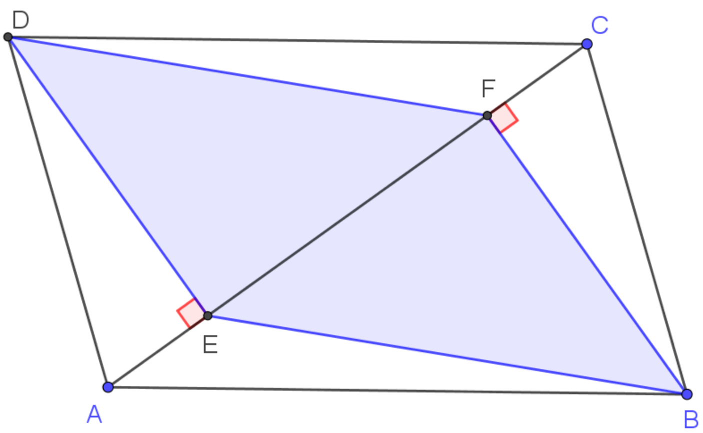
TASK 7: If ABCD is a parallelogram and AE = FC, prove that DEBF is also a parallelogram.



TASK 8: If AD is the bisector of angle A, DE parallel to AB, and EF parallel to BC, prove that the angles ABC and DEF are equal and AE = BF.



TASK 9: If ABCD is a parallelogram and DE, BF perpendicular to AC, prove that DEBF is also a parallelogram.



3rd Teaching hour: Evaluation – Feedback

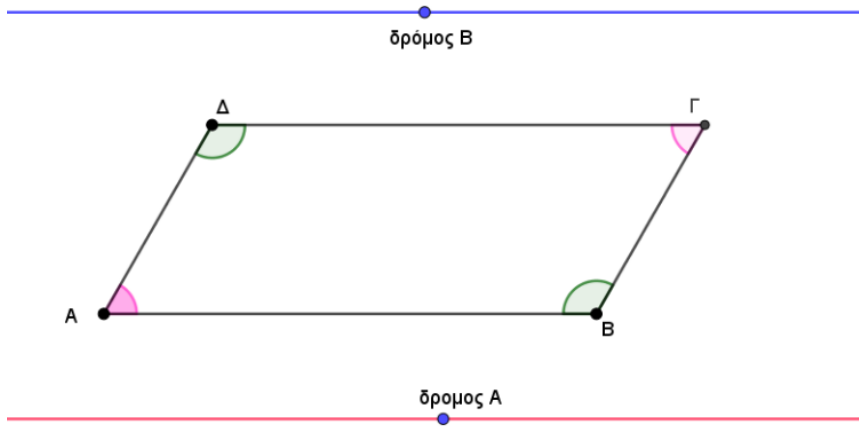
TASK 10

Two brothers inherited a plot of land in the shape of a parallelogram as shown in the figure. Help the siblings divide the plot fairly in each of the following cases:

a) Both siblings want access to both roads A and B.

b) One wants to have access only to road A and the other only to road B.

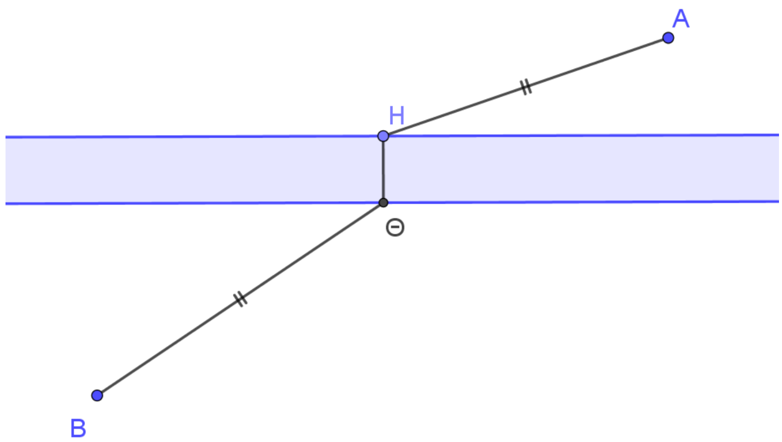
c) One wants to have access only to road A and the other only to road B and at the same time neither of them wants to build on the side streets on the side of which they agreed to become common areas.



TASK 11

Houses A and B are located on opposite banks of a river as shown in the diagram below. The community plans to build a bridge there on the condition that the residents of these houses also contribute financially and that the bridge is the same distance from the two houses. In what position should the engineer build the bridge?

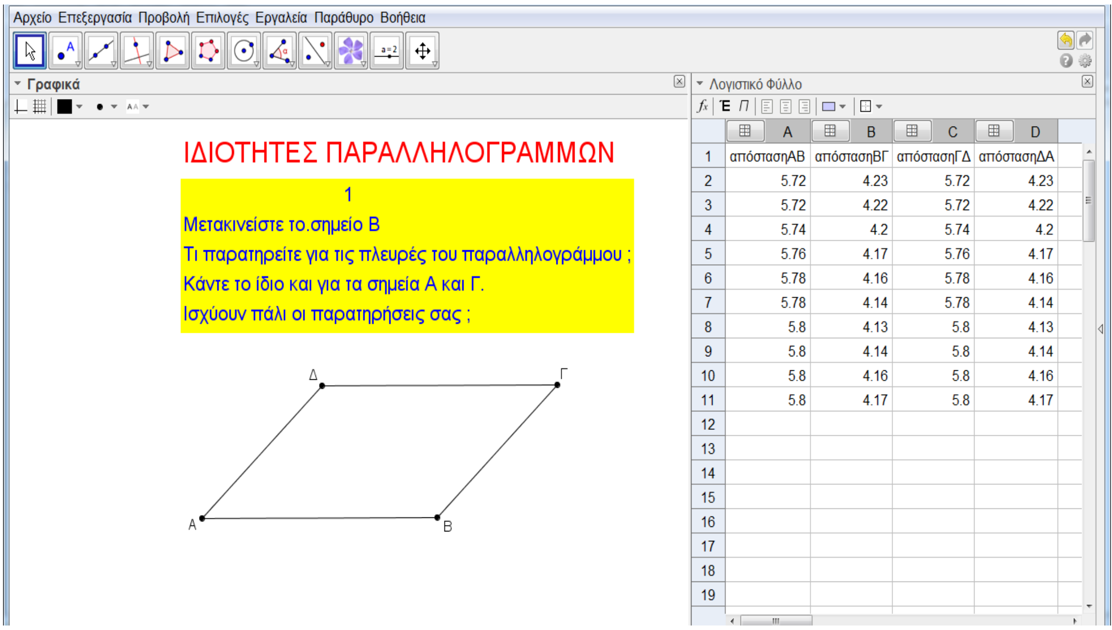
Justify your choices.



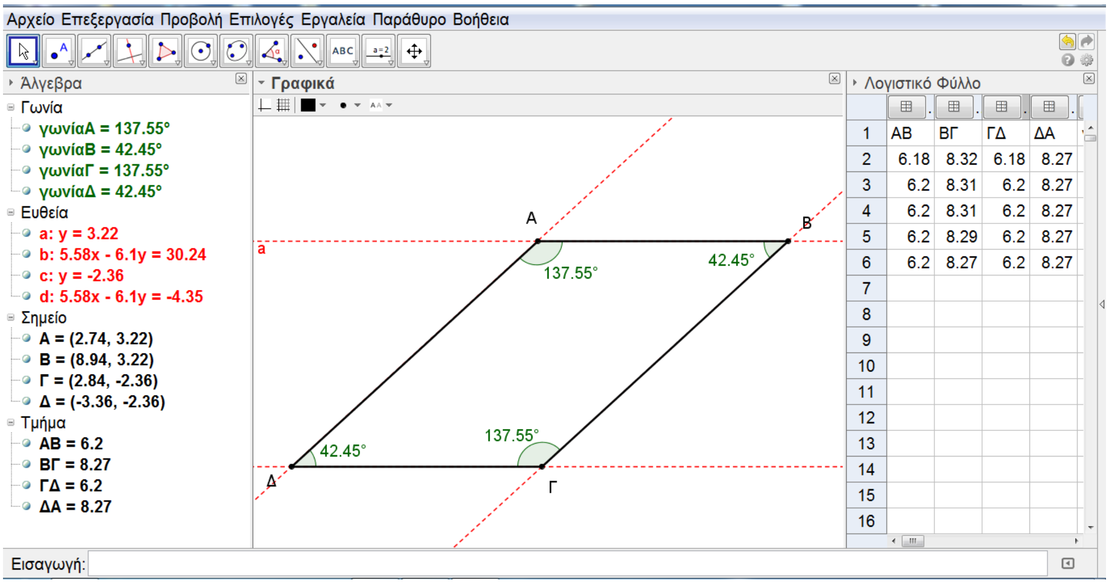
APPENDIX (List of alternative projects that can be used in the classroom)

- **To discover parallelogram properties and possible extension for special parallelograms.**

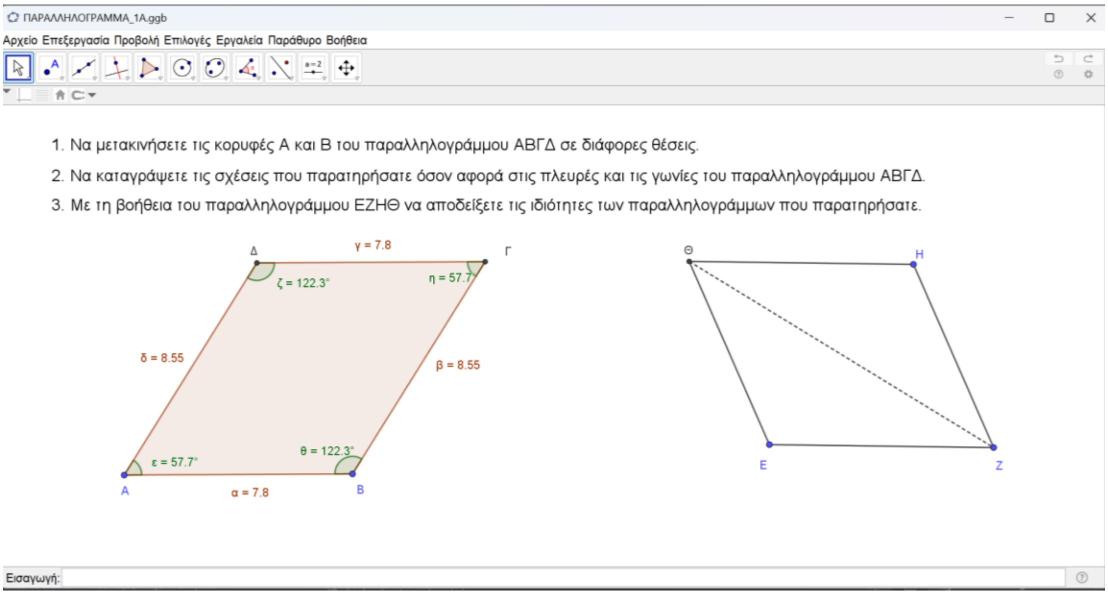
SNAPSHOT 1 (using software spreadsheet)



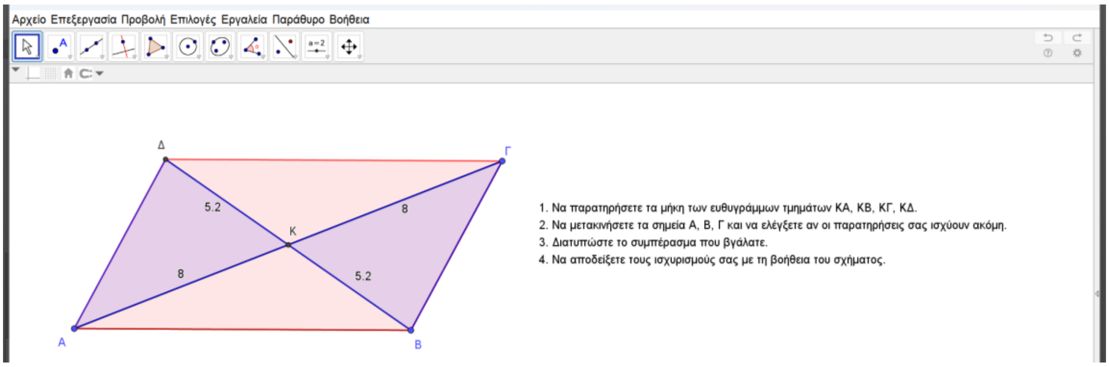
SNAPSHOT 2 (using the software's spreadsheet and Algebra's window)



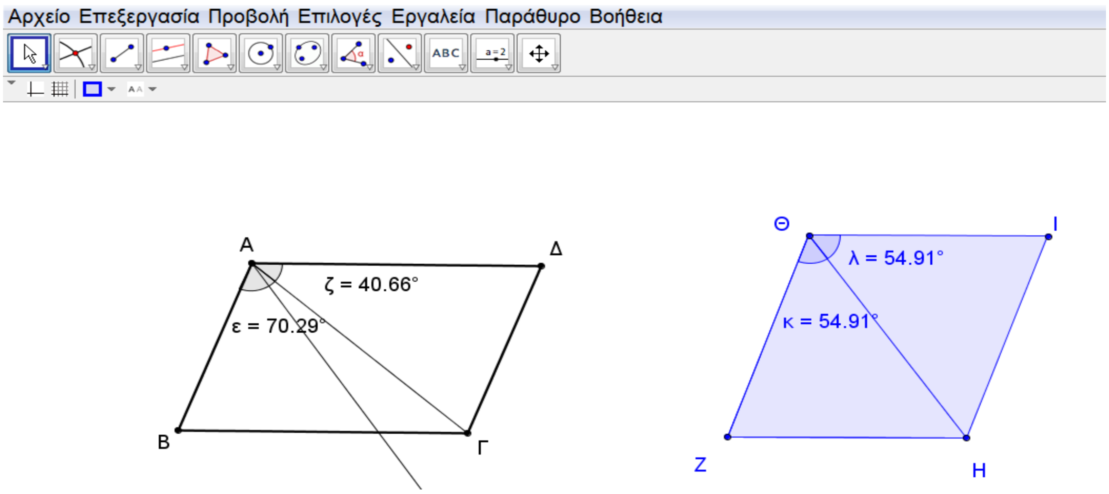
SNAPSHOT 3 (for properties of parallelogram angles and sides and proof)



SNAPSHOT 4 (for measurements, Property conjectures and their proof)

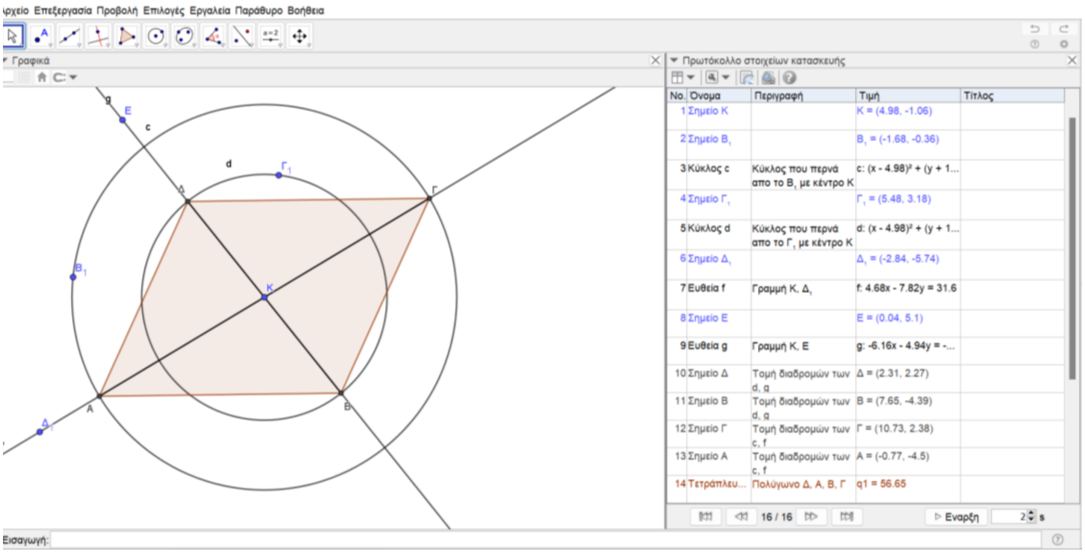


SNAPSHOT 5 (to remove the misconception that the diagonal is definitely also the bisector of its angle)

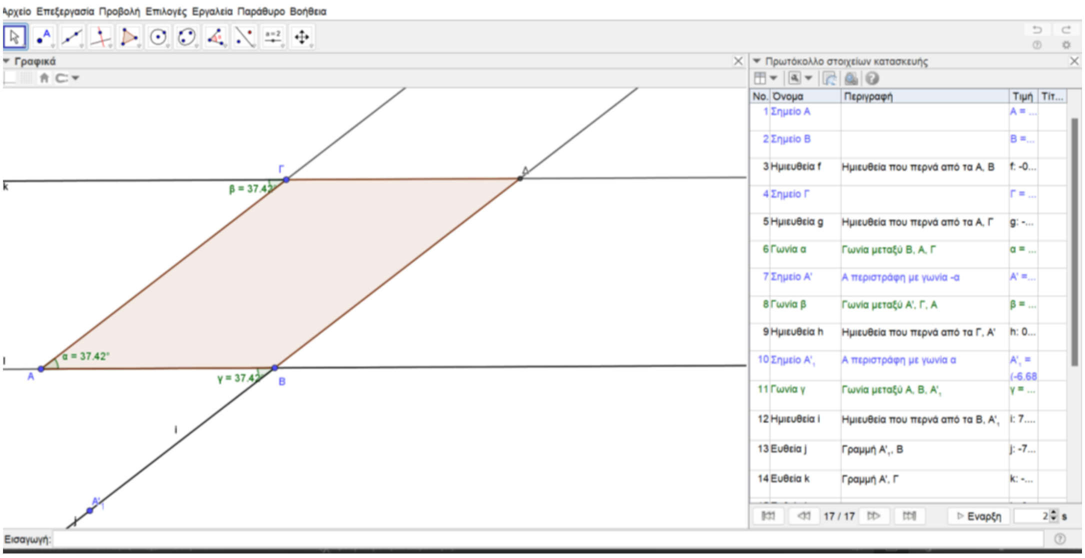


• **Constructions of parallelograms based on the criteria**

SNAPSHOT 1 (use build component protocol)

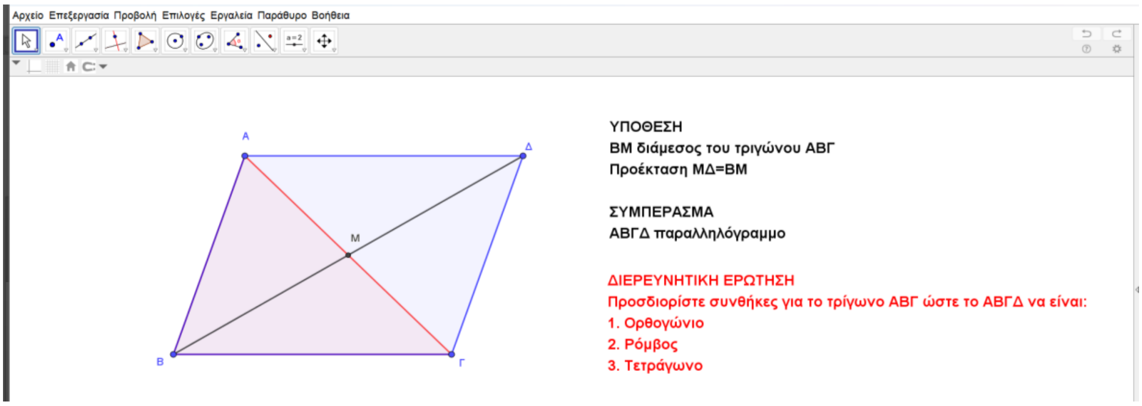


SNAPSHOT 2 (use build component protocol)

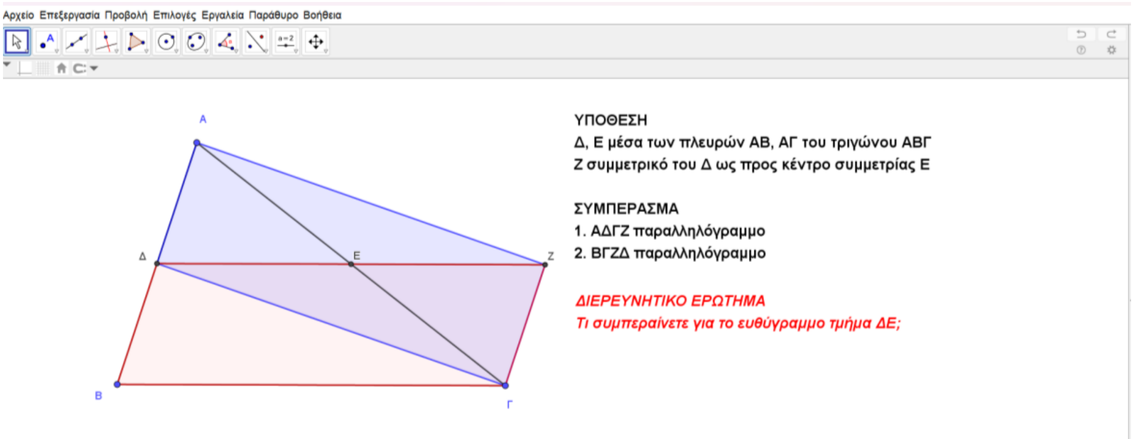


Extension also for special parallelograms, with or without using Geogebra.

SNAPSHOT 1

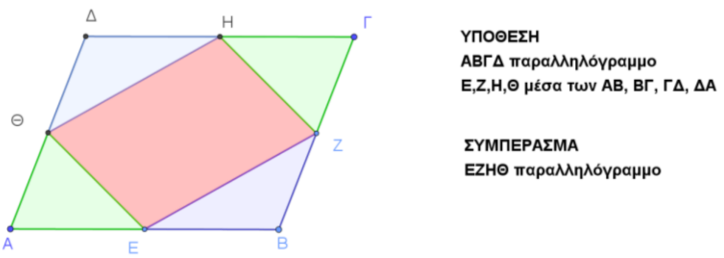


SNAPSHOT 2



TASK 12

Let ABCD be a parallelogram and let E, Z, H, Θ be the midpoints of the sides AB, ΒΓ, ΓΔ, ΔA respectively. Prove that the quadrilateral EZHΘ is a parallelogram.



Possible Extensions: This project can be extended to "Special Parallelograms" and "Applications of Parallelograms" and generalized to random quadrilateral ABCD (Parallelogram Theorem, Pierre Varignon).